## Josephson current in superconductor/ferromagnet/ superconductor junctions

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**Abstract.** The Bogoliubov-de Gennes equation and Nambu spinor Green's function approach are applied to studying the Josephson current in superconductor/ferromagnet/superconductor (S/F/S) Josephson junctions in the clean limit. It is found that the critical current exhibits a damped oscillation with the F thickness d, the oscillation period equal to  $2\pi\xi_F$  with  $\xi_F$  the coherence length of the F. The change of the critical current from positive to negative is determined by factor  $\cos \phi'$  with  $\phi' = d/\xi_F$  as the F-induced phase difference. The exponent decay of the critical current is close related to that of the superconductor order parameter in the F, both of them having the same decay length.

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It is well known that a supercurrent  $I_s$  could exist between two superconductors (Ss) separated by a thin insulating layer (I) in the absence of voltage drop between them, which is the so-called dc Josephson effect [1]. Owing to the quantum character of the superflow, a phase difference,  $\phi$ , between the superconductors appears. If the two superconducting electrodes of an S/I/S structure have different macroscopic phases,  $\phi_L$  and  $\phi_R$ , the current-phase relationship is given by  $I_s = I_c \sin(\phi_L - \phi_R) = I_c \sin \phi$  where  $I_c$  is the critical current. The Josephson effect also exists if two Ss are connected by a "weak link" of any physical nature (normal metal, semiconductor, geometrical constriction, etc.). The physical origin of the Josephson effect is the breakdown of time reversal symmetry in S/I/S structures due to the macroscopic phase difference between the two Ss. The physics of the dc Josephson effect with weak link can be understood by the Andreev reflection (AR) [2] processes of quasiparticles with energy smaller than the superconducting energy gap [3]. In the weak link region, an electron impinging on one of the interfaces is Andreev reflected and converted into a hole moving in the opposite direction, thus generating a Cooper pair in an S. This hole is consequently Andreev reflected at the second interface and is converted back to an electron, leading to the destruction of the Cooper pair in the other S. As a result of this cycle, a pair of correlated electrons is transferred from one S to another, creating a supercurrent flow across the junction [3]. The AR amplitudes depend on the corresponding phases  $\phi_L$  and  $\phi_R$ , and the resulting supercurrent depends on phase difference  $\phi$ .

The superconductor/ferromagnet (S/F) hybrid proximity structures and S/F-based multilayers have attracted much attention in experimental and theoretical investigations [3–29]. It is expected that similar characteristics of the Josephson effect exist in S/F/S junctions. Furthermore, there undoubtedly appear some particular effects in them, for carriers passing through the F must feel spindependent potentials as the result of the ferromagnetic exchange energy. The Andreev process, recognized as the mechanism of normal to supercurrent conversion [2,30], is modified at F/S interfaces due to the spin imbalance in the F. The current-carrying Andreev bound states are split and shifted in an oscillatory way under the influence of the F [31]. An electron and a hole with opposite spins and different momenta are correlated via Andreev reflection, thus providing an extension of superconducting order parameter into the F region of length of the order of  $\xi_F$ . Here  $\xi_F = \hbar v_F/2h_0$  is the coherence length in the F with exchange energy  $2h_0$  equal to the difference in energy between spin-up and spin-down bands, and  $v_F$ the Fermi velocity. It has been shown that inhomogeneous superconducting order parameter can be induced by the proximity effect in a thin F film in contact to an S [6] and in a weak F sandwiched between two Ss [7], even though  $h_0$  in the F is greater than  $\Delta_0$  in the S. A most interesting phenomenon in S/F/S junctions is the crossover from 0 to  $\pi$  state, which was observed in Nb/Cu<sub>x</sub>Ni<sub>1-x</sub>/Nb Josephson junctions by Ryazanov et al. [7]. The  $\pi$  state

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Fig. 1. Schematic illustration of the reflection and transmission processes of two types of quasiparticle injection processes at the S/F/S structure. The solid-line arrow stands for the electron with spin- $\sigma$  in F or electron-like quasiparticle in S, while the dotted-line arrow for the hole with spin- $\bar{\sigma}$  in F or hole-like quasiparticle in S.

is characterized by a phase shift of  $\pi$  in the ground state of a junction and is formally described by the negative critical current  $I_c(\phi')$  in the relation  $I_s = I_c(\phi') \sin \phi$  with  $\phi'$  the extra phase difference due to the F layer. Kontos et al. [11] investigated the Josephson coupling through a thin F film using S/I/F/S planar junctions and found damped-oscillatory behavior of  $I_c$  as a function of the F thickness.

In this paper we extend the theoretical approach of Blonder-Tinkham-Klapwijk (BTK) [30], which was previously used to study differential conductance of normalmetal (N)/S junction systems, to calculate wave functions of quasiparticles in an S/F/S structure. General expressions for both normal and Andreev reflection amplitudes are obtained, from which the Josephson current is calculated. Furthermore, we construct  $2 \times 2$  spinor retarded Green's function in the Nambu representation [32], with which the induced superconducting order parameter F(x)in the F is obtained. Inhomogeneous F(x) in the middle F layer can result in a phase difference  $\phi'$  across the junction, regardless of the original macroscopic phase difference  $\phi$ . This phase difference is given by  $\phi' = d/\xi_F$  with d the thickness of the F layer, and the critical current can be approximately expressed as  $I_c(\phi') = I_c \cos \phi'$  where  $I_c$ decreases monotonously with increasing d. The existence of  $\phi'$  leads to oscillations of Josephson current  $I_s$  with  $d/\xi_F$  and changes the sign of critical current  $I_c(\phi')$  at the crossovers between 0 and  $\pi$  states. In the clean limit, we obtain  $I_c$  decaying exponentially with  $d/\xi_F$ . Such a decaying  $I_c$  is close related to order parameter F(x) in the F, which decays with distance from the F/S interface. Both of them depend on the exchange energy, the F thickness, and temperature.

Consider an S/F/S Josephson junction consisting of two semi-infinite Ss and a ferromagnetic interlayer of thickness d. The F and Ss are separated by interfaces at x = 0 and x = d, as shown in Figure 1. The Ss are described by the BCS Hamiltonian, and their superconducting pair potentials are assumed to have the same magnitude but different phases ( $\phi_L$  and  $\phi_R$ ), given by  $\Delta(r) = \Delta(T)[\exp(i\phi_L)\Theta(-x) + \exp(i\phi_R)\Theta(x-d)]$ . Here

 $\varDelta(T)$  is the temperature dependent energy gap that follows the BCS relation  $\Delta(T) = \Delta_0 \tanh[1.76(T_c/T-1)]$ with  $T_c$  the critical temperature of the Ss, and  $\Theta(x)$  is the unit step function. The F layer is described by an effective single particle Hamiltonian with exchange energy  $2h_0$ , and each interface is described by a  $\delta$ -type potential with the same strength,  $V(x) = U\delta(x) + U\delta(x - d)$ . For simplicity, the effective masses m are taken to be equal in both F and S. We adopt the BdG approach [33] to study the S/F/S junction. This approach has been widely applied to describing quasiparticle states in superconductors with spatially varying pair potentials. In the F/S junction, the quasiparticle states are generally expressed by wave functions of four components, respectively, for electronlike quasiparticle (ELQ) and holelike quasiparticle (HLQ) with spin up and down. In the absence of spin-flip scattering, the four-component BdG equations may be decoupled into two sets of two-component equations: one for the spin-up electronlike and spin-down holelike quasiparticle wave functions  $(u_{\uparrow}, v_{\downarrow})$ , the other for  $(u_{\downarrow}, v_{\uparrow})$ . The BdG equation is given by

$$\begin{bmatrix} H_0(\mathbf{r}) - \eta_{\sigma} h(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -H_0(\mathbf{r}) - \eta_{\sigma} h(\mathbf{r}) \end{bmatrix} \begin{bmatrix} u_{\sigma}(x,\theta) \\ v_{\bar{\sigma}}(x,\theta) \end{bmatrix} = E \begin{bmatrix} u_{\sigma}(x,\theta) \\ v_{\bar{\sigma}}(x,\theta) \end{bmatrix}.$$
(1)

Here  $H_0(\mathbf{r}) = -\hbar^2 \nabla_r^2 / 2m + V(\mathbf{r}) - E_F$  with  $V(\mathbf{r})$  the usual static potential, E is the quasiparticle energy relative to Fermi energy  $E_F$ .  $h(\mathbf{r}) = h_0$  in the F region  $(0 \le x \le d)$ ,  $\eta_{\sigma} = 1$  for  $\sigma = \uparrow$  and -1 for  $\sigma = \downarrow$ , and  $\bar{\sigma}$  stands for the spin opposite to  $\sigma$ . Following the McMillan's method [34], we can obtain the reduced BdG equations for the quasiparticle wave functions. There are four types of quasiparticle injection processes in an S/F/S junction: an ELQ (HLQ) incident on the left (right) interface from the left (right) S. Let us consider two of them. Suppose a beam of spin- $\sigma$ ELQ incident on the interface at x = 0 at an angle  $\theta_S$  from the left S, as shown in Figure 1a. There are four possible trajectories in the Ss: ELQ  $(b_1^{\sigma})$  and HLQ  $(a_1^{\bar{\sigma}})$  reflections in the left S, transmissions to the right S as ELQ  $(c_1^{\sigma})$  and HLQ  $(d_1^{\bar{q}})$ . In the middle F layer there exist multireflected electrons  $(e_1^{\sigma} \text{ and } f_1^{\sigma})$  at angle  $\theta_A^+$  and holes  $(g_1^{\bar{q}} \text{ and } h_1^{\bar{\sigma}})$  at angle  $\theta_A^-$ . With general solutions of the BdG equation, the wave functions in S and F regions can be obtained. Owing to translational invariance in directions parallel to the interface, the wave functions in the S and F are given by

$$\Psi_{1\sigma}^{LS} = \begin{pmatrix} ue^{i\phi_L/2} \\ ve^{-i\phi_L/2} \end{pmatrix} e^{ik^+x\cos\theta_S} + a_1^{\bar{\sigma}} \begin{pmatrix} ve^{i\phi_L/2} \\ ue^{-i\phi_L/2} \end{pmatrix} e^{ik^-x\cos\theta_S} + b_1^{\sigma} \begin{pmatrix} ue^{i\phi_L/2} \\ ve^{-i\phi_L/2} \end{pmatrix} e^{-ik^+x\cos\theta_S}, \quad (2)$$

for  $x \leq 0$ ;

$$\Psi_{1\sigma}^{F} = \begin{pmatrix} 1\\0 \end{pmatrix} [e_{1}^{\sigma} e^{iq_{\sigma}^{+}x\cos\theta_{A}^{+}} + f_{1}^{\sigma} e^{-iq_{\sigma}^{+}x\cos\theta_{A}^{+}}] + \begin{pmatrix} 0\\1 \end{pmatrix} [g_{1}^{\bar{\sigma}} e^{iq_{\bar{\sigma}}^{-}x\cos\theta_{A}^{-}} + h_{1}^{\bar{\sigma}} e^{-iq_{\bar{\sigma}}^{-}x\cos\theta_{A}^{-}}], \quad (3)$$

for  $0 \le x \le d$ ; and

$$\Psi_{1\sigma}^{RS} = c_1^{\sigma} \begin{pmatrix} ue^{i\phi_R/2} \\ ve^{-i\phi_R/2} \end{pmatrix} e^{ik^+x\cos\theta_S} + d_1^{\bar{\sigma}} \begin{pmatrix} ve^{i\phi_R/2} \\ ue^{-i\phi_R/2} \end{pmatrix} e^{-ik^-x\cos\theta_S}, \quad (4)$$

for  $x \geq d$ , where  $u = \sqrt{(1 + \Omega/E)/2}$  and  $v = \sqrt{(1 - \Omega/E)/2}$  with  $\Omega = \sqrt{E^2 - \Delta^2(T)}$ . The wavevectors for the electrons and holes in the Ss are given by  $k^{\pm} = k_F \sqrt{1 \pm \Omega/E_F}$ , those in the F are given by  $q_{\sigma}^{\pm} = k_F \sqrt{1 \pm (\eta_{\sigma} h_0 \pm E)/E_F}$  with  $\sigma$  the spin index. In the BTK approach, since the wave-vector component parallel to the interface is assumed to remain unchanged in the reflection and transmission processes, the angles  $\theta_S$  and  $\theta_A^{\pm}$  differ from each other except when  $\theta_S = 0$ . All the coefficients  $a_1^{\bar{\sigma}}$ ,  $b_1^{\sigma}$ ,  $c_1^{\sigma}$ ,  $d_1^{\bar{\sigma}}$ ,  $e_1^{\sigma}$ ,  $f_1^{\sigma}$ ,  $g_1^{\bar{\sigma}}$  and  $h_1^{\bar{\tau}}$  can be determined by matching the boundary conditions:  $\Psi_{1\sigma}^{LS}(0) = \Psi_{1\sigma}^{F}(0)$  and  $(d\Psi_{1\sigma}^{F}/dx)_{x=0} - (d\Psi_{1\sigma}^{LS}/dx)_{x=0} = 2k_F \cos\theta_S Z \Psi_{1\sigma}^{F}(0)$  at x = 0, and  $\Psi_{1\sigma}^{F}(d) = \Psi_{1\sigma}^{RS}(d)$  and  $(d\Psi_{1\sigma}^{RS}/dx)_{x=d} - (d\Psi_{1\sigma}^{F}/dx)_{x=d} = 2k_F \cos\theta_S Z \Psi_{1\sigma}^{F}(d)$  at x = d, where  $Z = mU/k_F \cos\theta_S$  is a dimensionless parameter describing the magnitude of interfacial resistance.

For the injection of a beam of spin- $\bar{\sigma}$  HLQ on the interface at x = 0 at an angle  $\theta_S$  from the left S, as shown in Figure 1b, the wave functions are given by

$$\Psi_{2\bar{\sigma}}^{LS} = \begin{pmatrix} ve^{i\phi_L/2} \\ ue^{-i\phi_L/2} \end{pmatrix} e^{-ik^-x\cos\theta_S} + a_2^{\sigma} \begin{pmatrix} ue^{i\phi_L/2} \\ ve^{-i\phi_L/2} \end{pmatrix} e^{-ik^+x\cos\theta_S} + b_2^{\bar{\sigma}} \begin{pmatrix} ve^{i\phi_L/2} \\ ue^{-i\phi_L/2} \end{pmatrix} e^{ik^-x\cos\theta_S}$$
(5)

for 
$$x \leq 0$$
  

$$\Psi_{2\bar{\sigma}}^{F} = \begin{pmatrix} 0\\1 \end{pmatrix} [e_{2}^{\bar{\sigma}} e^{iq_{\bar{\sigma}}^{-}x\cos\theta_{A}^{-}} + f_{2}^{\bar{\sigma}} e^{-iq_{\bar{\sigma}}^{-}x\cos\theta_{A}^{-}}] \begin{pmatrix} 1\\0 \end{pmatrix} [g_{2}^{\sigma} e^{iq_{\sigma}^{+}x\cos\theta_{A}^{+}} + h_{2}^{\sigma} e^{-iq_{\sigma}^{+}x\cos\theta_{A}^{+}}]$$
(6)

for  $0 \le x \le d$ , and

$$\Psi_{2\bar{\sigma}}^{RS} = c_2^{\bar{\sigma}} \begin{pmatrix} v e^{i\phi_R/2} \\ u e^{-i\phi_R/2} \end{pmatrix} e^{-ik^- x\cos\theta_S} + d_2^{\sigma} \begin{pmatrix} u e^{i\phi_R/2} \\ v e^{-i\phi_R/2} \end{pmatrix} e^{ik^+ x\cos\theta_S}$$
(7)

for  $x \geq d$ . The wave functions for the other two types of quasiparticle injection processes can be obtained in a similar way. The dc Josephson current at a given temperature can be expressed by the Andreev reflection amplitudes in terms of the finite-temperature Green's function formalism [35]

$$I_{s} = \frac{e\Delta}{2\hbar} \sum_{k_{\parallel}} \sum_{\sigma,\omega_{n}} \frac{k_{B}T}{\Omega_{n}} [k^{+}(\omega_{n}) + k^{-}(\omega_{n})] \left[ \frac{a_{1}^{\bar{\sigma}}(i\omega_{n},\phi)}{k^{+}(\omega_{n})} - \frac{a_{2}^{\sigma}(i\omega_{n},\phi)}{k^{-}(\omega_{n})} \right]$$
(8)

where  $\omega_n = \pi k_B T(2n+1)$  are the Matsubara frequencies with  $n = 0, \pm 1, \pm 2, ...,$  and  $\Omega_n = \sqrt{\omega_n^2 + \Delta^2(T)}$ .  $k^+(\omega_n),$  $k^-(\omega_n), a_1^{\bar{\alpha}}(i\omega_n, \phi)$ , and  $a_2^{\sigma}(i\omega_n, \phi)$  are obtained from  $k^+,$  $k^-, a_1^{\bar{\alpha}}$  and  $a_2^{\sigma}$  by analytically continuing E to  $i\omega_n$ .

Next, we construct the Nambu spinor Green's function [32] in the S/F/S structure. With the wave functions  $\Psi_{i\sigma}(i = 1, 2, 3, 4 \text{ and } \sigma = \uparrow, \downarrow)$ , the retarded Green's function is given by [36,37]

$$G_{r}^{\sigma}(x, x', E) = \begin{cases} \alpha_{1}^{\sigma}\Psi_{3\sigma}(x)\Psi_{1\sigma}^{t}(x') + \alpha_{2}^{\sigma}\Psi_{3\sigma}(x)\Psi_{2\sigma}^{t}(x') \\ + \alpha_{3}^{\sigma}\Psi_{4\sigma}(x)\Psi_{1\sigma}^{t}(x') + \alpha_{4}^{\sigma}\Psi_{4\sigma}(x)\Psi_{2\sigma}^{t}(x'), \ x \le x' \\ \beta_{1}^{\sigma}\Psi_{1\sigma}(x)\Psi_{3\sigma}^{t}(x') + \beta_{2}^{\sigma}\Psi_{1\sigma}(x)\Psi_{4\sigma}^{t}(x') \\ + \beta_{3}^{\sigma}\Psi_{2\sigma}(x)\Psi_{3\sigma}^{t}(x') + \beta_{4}^{\sigma}\Psi_{2\sigma}(x)\Psi_{4\sigma}^{t}(x'), \ x \ge x' \end{cases}$$
(9)

where the wave function  $\Psi_{i\sigma}^{t}(x)$  is the transposition of  $\Psi_{i\sigma}(x)$ . The coefficients  $\alpha_{i}^{\sigma}$  and  $\beta_{i}^{\sigma}(i = 1, 2, 3, 4,)$ can be determined by satisfying the following boundary conditions:  $G_{r}^{\sigma}(x, x + 0_{+}, E) = G_{r}^{\sigma}(x, x - 0_{+}, E)$ , and  $dG_{r}^{\sigma}(x, x', E)/dx|_{x=x'+0_{+}} - dG_{r}^{\sigma}(x, x', E)/dx|_{x=x'-0_{+}} = (2m/\hbar^{2})\hat{\tau}_{3}$  with  $\hat{\tau}_{3}$  the Pauli matrix. After carrying out a little tedious calculation, we can get the 2×2 retarded Green's functions [38]. The superconducting order parameter F(x) is determined by the off-diagonal component of the Green's function with x = x',

$$F(x) = \frac{1}{\pi} \sum_{k_{\parallel},\sigma} \int_0^\infty dE Im[G_r^{\sigma}(x,x,k_{\parallel},E,\phi)]_{12}.$$
 (10)



Fig. 2. Josephson current  $I_s$  as a function of d for different  $h_0/E_F$ . Here Z = 0.3,  $\phi = \pi/2$ , and  $T = 0.2T_c$  are taken.

In what follows we discuss numerical results from equations (8) and (10). Figure 2 shows damped oscillations of Josephson current  $I_s$  as a function of the F thickness. Interestingly, the oscillation periods for different exchange energy are equal to  $2\pi\xi_F$ . With increasing  $h_0, \xi_F$  becomes short and does the oscillation period. Such a damped oscillation can be understood by the following argument. For an S/I/S structure with macroscopic phase difference  $\phi$ , the current-phase relationship is  $I_s = I_c \sin \phi$ . In the S/F/S junctions, the correlated electrons and holes in the F region have opposite spin directions and finite centerof-mass momentum  $Q = 1/\xi_F$  due to the existence of  $h_0$ . This leads to spatial dependent superconducting order parameters in the F [6, 38]. Such an oscillation of the order parameter is somewhat analogous to that of the "FFLO" state [39,40] in magnetic superconductors, but the pair potential is absent here. In our calculation it is found that the interference effect of the wave functions of electrons and holes in the F region results in oscillating factors such as  $\exp(ix/\xi_F)$ . As a pair of correlated electrons are transferred from one S to another via the F layer, a phase difference  $\phi' = d/\xi_F$  appears due to the oscillatory factor  $\exp(ix/\xi_F)$ . The critical current is approximately given by  $I_c(\phi') = I_c \cos \phi'$ . In Figure 2 the dc Josephson current changes the sign periodically according to  $\cos \phi'$  with  $\phi = \pi/2$ . In the absence of  $\phi'$  at d = 0, the junction is in the 0 state. With increasing  $d, \phi'$  increases and  $\cos \phi'$ changes its sign from positive to negative at  $d/\xi_F \approx \pi/2$ and back to positive at  $d/\xi_F \approx 3\pi/2$ . There appear periodic changes between 0 and  $\pi$  states with increasing d, in which the  $\pi$  state corresponds to the negative critical current. Another interesting result shown in Figure 2 is that rapid oscillations with small amplitude of the Josephson current are superimposed on oscillations related to the crossovers between 0 and  $\pi$  states. These rapid oscillations arise from coherent interference effects within the middle F layer in the clean limit [15].



Fig. 3. Phase-independent critical current  $I_c$  as a function of exchange energy  $h_0$  for d = 15 nm (solid line), 25 nm (dashed line) and 40 nm (dotted line). (b) Phase-independent critical current  $I_c$  as a function of d for  $h_0 = 0$  (solid line),  $0.05E_F$  (dashed line) and  $0.1E_F$  (dotted line).

Apparently, the phase-independent critical current  $I_c$ defined here is still a function of exchange energy  $h_0$ and F thickness d. As shown in Figure 3,  $I_c$  decreases monotonously either with increasing the exchange energy for fixed d or with increasing the F thickness for fixed  $h_0$ . It is found that curves in Figure 3 can be well fitted to  $I_c \propto \exp(-d/\xi_F)$ , the decay length just in agreement with the oscillation period. As a result, either an increase of d or a decrease of  $\xi_F = \hbar v_F/2h_0$  can equivalently result in an exponential decay of  $I_c$ . It is interesting to point out that this decaying behavior is obtained in the clean limit without introducing any impurity scattering. Such a decay of  $I_c$  is close related to the spatial variation of the superconducting order parameter in the F. Figure 4



Fig. 4. Spatial variation of the module of the superconducting order parameter in F for d = 25 nm,  $h_0/E_F = 0$ , 0.003, 0.01, 0.05 and 0.1 (a); and  $h_0/E_F = 0.05$ , d = 10, 15 and 25 nm (b).

shows calculated results for the spatial variation of |F(x)|, the module of the superconducting order parameter, in the middle F layer of an S/F/S Josephson junction. The order parameter is diminished with distance from the F/S interface and exhibits a minimum at the center of the F layer. The finite F(x) in the F is a result of interference of the correlated electrons and holes in the F region, and stems from the proximity effect of the superconducting electrodes. It then follows that the decrease of  $I_c$  and |F(x)| in the F is of the same origin, the F-induced loss of the coherence of electrons and holes in the Andreev bound states [41]. With increasing either  $h_0$  or d, the coherencebroken effect is enhanced, producing a decrease in |F(x)|and  $I_c$ .

In summary, we have studied the dc Josephson current  $I_s$  of the S/F/S junctions in the clean limit using the BdG equation and Nambu spinor Green's function approach. It is found that as a pair of correlated electrons are trans-

ferred from one S to another via the F layer, there appears an extra phase difference  $\phi' = d/\xi_F$  induced by the exchange energy of the F layer. This leads to a damped oscillation of critical current  $I_c(\phi')$  with the F thickness from the 0 to  $\pi$  states, the oscillation period equal to  $2\pi\xi_F$ . Owing to the existence of the ferromagnetic exchange energy, the superconducting order parameter in F decays exponentially, resulting in an exponential decay of  $I_c$  of the S/F/S junction with decay length  $\xi_F$ . Numerical results indicate that, for an S/F/S junction with low interfacial transparency, the Josephson current can be approximately expressed as  $I_s = I_c(0) \exp(-d/\xi_F) \cos \phi' \sin \phi$  with  $I_c(0)$  constant if the coherent (geometrical) oscillations with small amplitude are neglected.

Most of the practical S/F/S junctions are in the dirty limit. The present results in the clean limit are qualitatively consistent with those in the dirty limit. In the ballistic junctions, however, spatial oscillations are easier to observe and the transition region of coexisting 0 and  $\pi$ states is larger. In addition, the coherency effects result in rapid oscillations with small amplitude of the Josephson current superimposed on oscillations related to the crossovers between 0 and  $\pi$  states. The  $\pi$  junction may be used as the phase inverter in superconducting digital circuits, and for engineering a quantum two-level system, or qubit, which is the basic element of a quantum computer.

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